IV. Example Application

Distances and positions are measured in nautical miles from a fixed origin in the xy plane. Steering (heading) angles χ are measured in degrees, anticlockwise from the positive x axis. The units of time and speed are hours and knots, respectively. The maximum allowable turn rates u_{Mi} are given in degrees per second and are consistent with standard rate turns at the speeds chosen for this example. To provide a basis for comparison, the example conflict configuration chosen here is same configuration as that considered in Ref. 9, except that, in that study, only one aircraft executed the avoidance maneuver at a time. The data for the example conflict avoidance configuration are shown in example 1:

$$\{(a_1, b_1), (a_2, b_2), \chi_{1o}, \chi_{2o}, v_1, v_2, u_{M1}, u_{M2}, r_s\} = \{(0, 0), (10, -8), 0.0, 97.15, 200, 250, 2.86, 2.86, 5\} \quad (13)$$

The output required for the optimal control and display of the conflict resolution maneuvers are the switching times $t^*_{i,j}$, which were computed to within an error tolerance of 0.25 s for each feasible extremal using $\Delta t = 0.25$ s. In example 1, the optimal solution is the RR feasible extremal requiring 207.25 s to execute. The switching times were computed to be (in seconds)

$$t^*_{1,1} = 5.75, \qquad t^*_{2,1} = 5.25, \qquad t^*_{1,2} = t^*_{2,2} = 126.5$$
$$t^*_{1,3} = t^*_{2,3} = 191.0, \qquad t^*_{1,f} = t^*_{2,f} = 207.25$$

Only one iteration of step 5 in the algorithm was necessary. Because no final radials were required for the optimal extremals (i.e., $t^*_{i,4} = t^*_{i,3}$, $i = 1, 2$ in step 4 of the algorithm), t^*_4 is not included here. The plots of the optimal trajectories are given in Fig. 1a. Figure 1b illustrates the 64.5 s of boundary arc control. A comparison of the results for example 1 considered here with those obtained for (the same initial configuration) examples 1c and 1d in Ref. 9 show that permitting simultaneous cooperative maneuvering can reduce path deviation times considerably (in this case, to 207.25 s from 254.1 s), can yield entirely different optimal trajectories (cf. Ref. 9, where the initial left turn by aircraft 2 proved optimal), and can substantially reduce the extent of the total deviation from the nominal ground tracks. Finally, numerical experiments were carried out to verify that the algorithm derived here (with the additional constraint that only one aircraft executes the avoidance maneuver) reproduces the analytical results obtained for example 1 in Ref. 9.

V. Conclusions

In Sec. III, a simple time-stepping algorithm was presented that will detect potential conflicts and resolve them by computing the globally optimal steering programs for both aircraft in real time. The algorithm is robust because at each time step the procedure verifies that the separation constraint is satisfied. Furthermore, the algorithm accurately recovers all of the boundary arc control history, which, as is shown in the example application in Sec. IV, may constitute a significant portion of the optimal avoidance maneuvers. The algorithm was generated using the form of the optimal control law determined from an analysis of the associated Hamiltonian and costate equations. Incorporating multiple conflicts, more general performance measures, as well as variable airspeeds, into the model will be an important consideration for future work. Because of its simplicity, the algorithm derived may prove to be a useful tool in exploring the design of a comprehensive conflict detection and avoidance system.

Acknowledgment

This research was supported by the Natural Sciences and Engineering Research Council of Canada.

References

[1] Lanzer, N., and Jenny, M. T., "Managing the Evolution to Free Flight," *Journal of Air Traffic Control*, 1995, pp. 8–20.

[2] Erzberger, H., "Design Principles and Algorithms for Automated Air Traffic Management," Knowledge-Based Functions in Aerospace Systems, LS-200, AGARD, Nov. 1995.

[3] Denery, D. G., Erzberger, H., Davis, T. J., Green, S. M., and McNally, B. D., "Challenges of Air Traffic Management Research: Analysis, Simulation, and Field Test," AIAA Paper 97-3832, Aug. 1997.

[4] Krozel, J., Mueller, T., and Hunter, G., "Free-Flight Detection and Resolution Analysis," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, AIAA, Reston, VA, 1996, pp. 1–11.

[5] Tomlin, C., Pappas, G., and Sastry, S., "Conflict Resolution in Air Traffic Management: A Study in Multi-Agent Hybrid Systems," *IEEE Transactions on Automatic Control*, Vol. 43, No. 4, 1998, pp. 509–521.

[6] Zhao, Y. J., and Schultz, R. L., "Deterministic Resolution of Two-Aircraft Conflict in Free Flight," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, AIAA, Reston, VA, 1997, pp. 469–478.

[7] Menon, P. K., Sweriduk, G. D., and Sridhar, B., "Optimal Strategies for Free-Flight Air Traffic Conflict Resolution," *Journal of Guidance, Control, and Dynamics*, Vol. 33, No. 2, 1999, pp. 202–211.

[8] Kuchar, K. K., and Yang, C. Y., "Survey of Conflict Detection and Resolution Modeling Methods," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, AIAA, Reston, VA, 1997, pp. 469–478.

[9] Clements, J. C., and Ingalls, B., "An Extended Model for Pairwise Conflict Resolution in Air Traffic Management," *Optimal Control Applications and Methods*, Vol. 20, 1999, pp. 183–197.

[10] Hocking, L. M., *Optimal Control: An Introduction to the Theory with Applications*, Oxford Univ. Press, New York, 1991.

Mechanism for Precision Orbit Control with Applications to Formation Keeping

I. Michael Ross*
Naval Postgraduate School,
Monterey, California 93943

Introduction

DETECTING gravitational waves requires a very large baseline (from hundreds to a million kilometers) for an interferometer.[1,2] Challenged by this extraordinary requirement, a number of astrophysicists have explored various ways to achieve this from an engineering viewpoint. As documented in Refs. 1 and 2, the idea of using three separated spacecraft orbiting in formation to achieve effective structural rigidity (required by the baseline of the interferometer) was suggested in the early 1980s by Faller and Bender. Based on this concept, the laser interferometer space antenna (LISA) project was proposed to ESA in 1993, resulting in an ESA/NASA project.[1,2] When the impact of such possible configurations for terrestrial application was recognized, the U.S. Air Force declared in 1995 that a distributed spacecraft system (DSS) flying in formation was one of the key technology areas for the 21st century.[3] It is apparent that, since then, a great deal of research has been done on DSS and formation keeping. Of particular note are Refs. 4–6 and the references contained therein. Carter[4] presents an excellent survey of linearized equations for relative motion between elliptical orbits. Recently, Melton[5] extended the equations for a state transition matrix by way of a truncated power series in eccentricity. A perspective on these and other analyses as it pertains to formation keeping is presented by Vadali et al.[6]

One of the central technology areas for formation keeping is maintaining the DSS in formation with little or no propellant. In addition, for interferometric measurements, it is necessary to sense and control the distributed system to an effective structural tolerance of the order of a fraction of the wavelength of interest. In this Note, we propose a simple nonpropulsive mechanism to control the orbital position of a spacecraft to such a high precision. Simply stated, the

Received 6 February 2001; revision received 19 November 2001; accepted for publication 20 February 2002.

*Associate Professor, Department of Aeronautics and Astronautics, Code: AA/Ro; imross@nps.navy.mil. Associate Fellow AIAA.

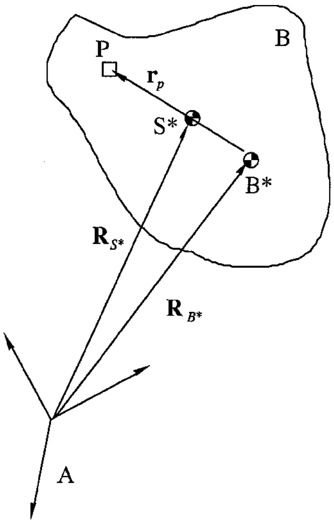

Fig. 1 Description of various vectors.

method is based on repositioning small masses attached to a spacecraft. Although this idea is quite simple, it has apparently not been proposed for precision orbit control.

Physical Concept

Let A be an arbitrary reference frame (not necessarily Newtonian), B a rigid body, and P a point mass (Fig. 1) that is movable inside (or outside) B. In the equations to follow in this section, if P is not a point mass (as in a practical situation), then all of the assertions will hold if P is replaced by an equivalent mass at its c.m., P^*. Let $\boldsymbol{r}_p$ be the position vector of P from B^*, the c.m. of B. The vector of interest is $\boldsymbol{R}_{B^*}$, the position vector of B^* from the origin of frame A. Let $\boldsymbol{R}_{S^*}$ be the position vector of S^* from the origin of A, where S is the system $S = B + P$ and S^* is the c.m. of S. By the definition of a c.m., we have

$$\boldsymbol{R}_{S^*} = \boldsymbol{R}_{B^*} + \mu_R \boldsymbol{r}_P \tag{1}$$

where μ_R is the mass ratio defined as

$$\mu_R = m/(M + m) \tag{2}$$

where M and m are the masses of B and P, respectively.

It is evident from Eq. (1) that the c.m. of B can be controlled to a second-order accuracy by a first-order movement of the particle P. In other words, suppose we let $\boldsymbol{R}_{S^*} = \boldsymbol{0}$ (by moving the origin of frame A to S^*), then the motion of B^* in A is given by $\boldsymbol{R}_{B^*} = -\mu_R \boldsymbol{r}_P \Rightarrow \delta\boldsymbol{R}_{B^*} = -\mu_R \delta\boldsymbol{r}_P \sim \mathcal{O}(\varepsilon^2)$ if $\mu_R \sim \mathcal{O}(\varepsilon)$ and $\delta\boldsymbol{r}_P \sim \mathcal{O}(\varepsilon)$. Thus, for example, if we move a particle of mass as large as 1% over a distance of 1 cm, the motion of B^* (in A) is as small as a $\frac{1}{10}$ of a millimeter or 100 μm. Thus, control of the position of an object at infrared wavelengths is easily attainable with such a crude mechanism. It is obvious that control in the visible region of the electromagnetic spectrum can be obtained by moving a smaller mass, for example, 0.1%, over a smaller distance, for example, 1 mm.

Application to Spacecraft Formation Keeping

By definition, two bodies are said to be in formation if the distance between their c.m. is a constant. If the distances between their c.m. is not a constant, then it is necessarily bounded for orbital eccentricities of less than one. Hill's equations (see Ref. 7) for the perturbed motion of the c.m. of a spacecraft ($S = B + P$) are given by

$$\ddot{X}_S - 2n\dot{Y}_S - 3n^2 X_S = a_x \tag{3a}$$

$$\ddot{Y}_s + 2n\dot{X}_s = a_y \tag{3b}$$

$$\ddot{Z}_s + n^2 Z_s = a_z \tag{3c}$$

where X_S, Y_S, and Z_S are the usual components of $\boldsymbol{R}_{S^*}$ in Hill's orbital frame. Note that we retain the perturbing acceleration terms in Eqs. (3a–3c); hence, in principle, our formulation allows us to include perturbation effects arising from J_2, atmosphere, etc. In any event, substituting Eq. (1) in Eqs. (3a–3c), we get

$$\ddot{X} - 2n\dot{Y} - 3n^2 X + \mu_R(\ddot{x} - 2n\dot{y} - 3n^2 x) = a_x \tag{4a}$$

$$\ddot{Y} + 2n\dot{X} + \mu_R(\ddot{y} + 2n\dot{x}) = a_y \tag{4b}$$

$$\ddot{Z} + n^2 Z + \mu_R(\ddot{z} + n^2 z) = a_z \tag{4c}$$

where X, Y, and Z and x, y, and z are the components of $\boldsymbol{R}_{B^*}$ and $\boldsymbol{r}_p$ in Hill's frame, respectively. Define an arbitrary configuration C by

$$X \equiv X_c(t) \tag{5a}$$

$$Y \equiv Y_c(t) \tag{5b}$$

$$Z \equiv Z_c(t) \tag{5c}$$

The proof mass dynamics then obeys the laws

$$\ddot{x} - 2n\dot{y} - 3n^2 x = (a_x - C_x)/\mu_R \tag{6a}$$

$$\ddot{y} + 2n\dot{x} = (a_y - C_y)/\mu_R \tag{6b}$$

$$\ddot{z} + n^2 z = (a_z - C_y)/\mu_R \tag{6c}$$

where C_x, C_y, and C_z are the configuration dynamics stipulated by

$$C_x = \ddot{X} - 2n\dot{Y} - 3n^2 X \tag{7a}$$

$$C_y = \ddot{Y} + 2n\dot{X} \tag{7b}$$

$$C_z = \ddot{Z} + n^2 Z \tag{7c}$$

Thus, in principle, any configuration may be achieved; however, in practice this concept is limited by the requirement that $\boldsymbol{r}_p$: (x, y, z) be bounded by the characteristic length of the spacecraft. Hence, mechanisms based on this principle are better suited for precision control than gross control. The order of precision that can be achieved for a given configuration C is determined by the order of accuracy that can be obtained in implementing Eq. (6). In any case, one can view the physics of the mechanism in terms of a proof mass that compensates for all of the undesired motion. The compensation is achieved by way of the action–reaction force between the body B and the mass P.

Design of Possible Mechanisms

We describe some possible designs of the mechanism described in the preceding section. One simple design is to locate three masses along three orthogonal (or linearly independent) control axes that guide the masses along a straight line relative to the body. Adding such a device on both sides of the c.m. of the main body B of the spacecraft might be beneficial in not only providing range of motion but also redundancy. The masses may be designed to slide or perform a screw motion along the guide. The advantage of a screw motion is that it allows a natural way to characterize the relative location of the proof mass by way of the screw gauge. The screw calibration may be used as an addition input to calibrate the position of the mass. Another simple idea is to have larger masses move over possibly larger distances for crude control and a similar system with smaller masses for finer control. Finally, one could always use a thruster for coarse control and a mass movement mechanism for fine control, thus alleviating propellant and thruster requirements (such as the minimum impulse-bit requirement for fine control).

Attitude Control Issues

By definition, moving a mass changes the inertia dyadic of the system S as well as its angular momentum. The purpose of this section is simply to analyze and quantify these changes so that they can be taken into account in an attitude control system.

To this end, consider $\boldsymbol{I}^{S/S^*}$, the central inertia dyadic of the system S. It is straightforward to show that[8]

$$\boldsymbol{I}^{S/S^*} = \boldsymbol{I}^{B/B^*} + \boldsymbol{I}^{P/P^*} + [Mm/(M + m)](\boldsymbol{r}_P \cdot \boldsymbol{r}_P \boldsymbol{U} - \boldsymbol{r}_P \boldsymbol{r}_P) \tag{8}$$

where $\boldsymbol{I}^{B/B^*}$ and $\boldsymbol{I}^{P/P^*}$ are the constant central inertia dyadics of the rigid body B and the mass P and $\boldsymbol{U}$ is the unit dyad. Thus, the first-order change in the central inertia dyadic of the system $\delta\boldsymbol{I}^{S/S^*}$ is given by

$$\delta \boldsymbol{I}^{S/S^*} = [Mm/(M+m)](2\delta \boldsymbol{r}_P \cdot \boldsymbol{r}_P \boldsymbol{U} - \delta \boldsymbol{r}_P \boldsymbol{r}_P - \boldsymbol{r}_P \delta \boldsymbol{r}_P) \tag{9}$$

The order of magnitude of this change can be expressed more succinctly by letting R_G be the radius of gyration of B. Thus, we can express $\|\delta \boldsymbol{I}^{S/S^*}\|$ as

$$\left\| \frac{\delta \boldsymbol{I}^{S/S^*}}{\boldsymbol{I}^{S/S^*}} \right\| \sim \mu_R \left(\frac{\delta r_P}{R_G}\right)\left(\frac{r_P}{R_G}\right) \tag{10}$$

which shows that the fractional change in the inertia is a second-order effect and may even be reduced to a third-order effect by keeping $r_P/R_G \sim \mathcal{O}(\varepsilon)$. Similar to the inertia dyadic, an expression for the central angular momentum may be written as

$$\boldsymbol{H}^{S/S^*} = \boldsymbol{I}^{S/S^*} \cdot {}^N\boldsymbol{\omega}^B + \boldsymbol{I}^{P/P^*} \cdot {}^B\boldsymbol{\omega}^P + [mM/(M+m)]\boldsymbol{r}_P \times {}^B\boldsymbol{V}^{P^*} \tag{11}$$

where ${}^N\boldsymbol{\omega}^B$ is the angular velocity of B with respect to N, a Newtonian frame, and ${}^B\boldsymbol{V}^{P^*}$ is the velocity of P^* with respect to B. The last two terms of this equation may be viewed as extra terms in the original angular momentum. Because these are either precisely known quantities, for example, ${}^B\boldsymbol{\omega}^P = \boldsymbol{0}$ for a sliding proof mass, or quantities whose effects can be calibrated, it can be taken into account without much difficulty in a precision attitude control system. Note also that for precision control, we may require ${}^BV^{P^*} \sim \mathcal{O}(\varepsilon)$ or perhaps even ${}^BV^{P^*} \sim \mathcal{O}(\varepsilon^n) n > 1$ so that the last term is Eq. (11) can be made quite small. In any case, for the precision orbital maneuvering system proposed in this Note, an attitude control system that accounts for the orbit-attitude coupling is necessary.

Conclusions

In any mechanism that purports precision control, there are the usual issues in precision manufacturing. Our mechanism is no different in this context. However, the two main features of our mechanism are 1) the possibility to achieve orbit control without the use of mass expulsion and 2) the ability to achieve second-order precision in the state vector from a first-order precision in control. One limitation of this mechanism is that it can be used only for periodic disturbances that generate perturbations that are no larger than the characteristic length of the spacecraft. For perturbations that are secular, the mechanism can still be used, but periodic reinitialization, similar to the practice of momentum dumping in attitude control, is necessary. Further work on how this mechanism couples with the attitude control system is warranted.

Acknowledgments

I would like to thank Tom Thorvaldsen and Matt Bottkol, both of Charles Stark Draper Laboratories, for encouraging me to pursue the ideas presented here.

References

[1]Bender, P., Brillet, A., Ciufolini, I., Cruise, A. M., Cutler, C., Danzmann, K., Fidecaro, F., Folkner, W. M., Hough, J., McNamara, P., Peterseim, M., Robertson, D., Rodrigues, M., Rüdiger, A., Sandford, M., Schäfer, G., Schilling, R., Schutz, B., Speake, C., Stebbins, R. T., Sumner, T., Touboul, P., Vinet, J.-Y., Vitale, S., Ward, H., and Winkler, W., "LISA: Laser Interferometer Space Antenna, Pre-Phase A Report," Max-Planck-Inst., Rept. MPQ 208, Garching, Germany, Dec. 1995.

[2]Bender, P., Ciufolini, I., Danzmann, K., Folkner, W. M., Hough, J., Robertson, D., Rodrigues, M., Rüdiger, A., Sandford, M., Schilling, R., Schutz, B., Stebbins, R. T., Sumner, T., Touboul, P., Vitale, S., Ward, H., and Winkler, W., "LISA: Laser Interferometer Space Antenna, Pre-Phase A Report," 2nd ed., Jet Propulsion Lab., JPL Rept. MPQ 233, California Inst. of Technology, Pasadena, CA, July 1998.

[3]*New World Vistas*, Summary Vol., U.S. Air Force Scientific Advisory Board, Dec. 1995.

[4]Carter, T. E., "State Transition Matrices for Terminal Rendezvous Studies: Brief Survey and New Examples," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 1, 1998, pp. 148–155.

[5]Melton, R. G., "Time-Explicit Representation of Relative Motion Between Elliptical Orbits," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 4, 2000, pp. 604–610.

[6]Vadali, S. R., Alfriend, K. T., and Vaddi, S. S., "A New Concept for Constellation Control of Formation Flying Satellites," American Astronautical Society, Paper AAS 01-218, Feb. 2001.

[7]Kaplan, M. H., *Modern Spacecraft Dynamics and Control*, Wiley, New York, 1976, p. 111.

[8]Kane, T. R., Likins, P. W., and Levinson, D. A., *Spacecraft Dynamics*, McGraw–Hill, New York, 1983, p. 213.

Consistent Approximations to Aircraft Longitudinal Modes

N. Ananthkrishnan* and P. Ramadevi†
Indian Institute of Technology Bombay, Mumbai 400 076, India

I. Introduction

EVER since Lanchester[1] derived an approximation to the phugoid frequency, literal approximations have formed an important part of the theory of aircraft dynamic stability.[2–4] However, the standard phugoid approximation often fails to match well with aircraft data, whereas the short period approximation is usually found to be numerically accurate. This has led to the derivation of several alternate phugoid approximations, many of which have been reviewed by Pradeep.[5] Unfortunately, the relative merits of the competing phugoid approximations could only be compared by matching them with numerical data, and none of them proved to be uniformly satisfactory. Part of this mystery has been resolved in a recent work,[6] which uncovered a serious flaw in previous derivations that led to incorrect approximations to the slow modes in general, and the phugoid mode in particular. Meanwhile, nearly a century after Lanchester, researchers are still engaged in deriving improved approximations to the phugoid mode.[5,7] Pradeep[5] appears to have been the first to realize that "correct" approximations to the longitudinal modes should satisfy the fourth-order longitudinal characteristic polynomial. Then, there would be no need to test this approximation for its correctness against numerical data, nor could this approximation be improved upon in any manner. Believing the existing short period approximation to be correct, Pradeep attempted to derive a correct phugoid approximation by factoring out the second-order short period polynomial from the fourth-order characteristic polynomial. However, his formulation had just two unknowns, which could not satisfy all four equations for the coefficients of the characteristic polynomial.

Rather than start with the characteristic polynomial, in the present Note we follow a formal procedure to derive systematically correct literal approximations to the aircraft longitudinal modes. No assumption is made except that there exist two distinct timescales corresponding to the fact that the short period mode is notably faster than the phugoid for most conventional airplanes. The ratio of the two timescales, which is small, is then exploited as an order parameter, and approximations are derived to zeroth- and first-order in the small parameter. These approximations are consistent in the sense that they retain all terms up to a certain order in the small parameter, thus ensuring that there are no missing terms in the approximations at each order. The consistent approximations are shown to match all of the coefficients of the characteristic polynomial perfectly at each order. The present derivation is the first instance where the literal approximations have been shown to check with all of the coefficients of the characteristic polynomial. In particular, the existing short period approximation is shown to be not consistent at the first order, which partly explains Pradeep's failure to derive a correct phugoid approximation by factorization.

Received 7 May 2001; revision received 1 March 2002; accepted for publication 7 March 2002.

*Assistant Professor, Department of Aerospace Engineering; akn@aero.iitb.ac.in. Senior Member AIAA.

†Assistant Professor, Department of Physics; ramadevi@phy.iitb.ac.in.